iDASH - Secure Genome Analysis Competition Using ObliVM

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Programming Framework for Secure Computation

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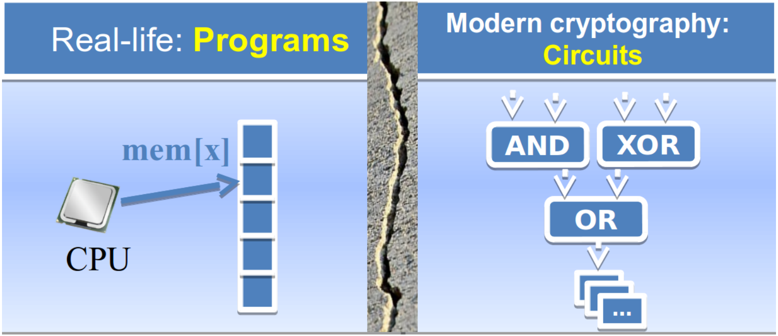
ObliVM
Task 1A
Task 1B
Set union
Task 2A
Task 2B



Programming Framework for Secure Computation

Ease-of-use: easy for non-specialist programmers to use

Efficiency: compiles programs to *small* circuits

Formal Security: type system is being formalized

http://oblivm.com

Compute MAF

- Compute minor allele frequencies

Alice	**Bob**
AA AC AA	AA AC CC
$f_A^{Alice} = 5, f_C^{Alice} = 1$	$f_A^{Bob} = 3, f_C^{Bob} = 3$

Cleartext
Secure

Compute MAF

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Compute $min(f_A^{Alice} + f_A^{Bob}, f_C^{Alice} + f_C^{Bob})$

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Compute $min(f_A^{Alice} + f_A^{Bob}, f_C^{Alice} + f_C^{Bob})$

Secure Computation: $MAF = min(f_A^{Alice} + f_A^{Bob}, f_C^{Alice} + f_C^{Bob})$

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Code in ObliVM-lang: Compute MAF


```
struct Task1aAutomated@m@n{};
void Task1aAutomated@m@n.funct(int@m[public n] alice_data,
                               int@m[public n] bob_data,
                               int@m[public n] ret,
                               public int@m total_instances) {
  int@m total = total_instances;
  int@m half = total_instances / 2;
  for (public int32 i = 0; i < n; i = i + 1) {
    ret[i] = alice_data[i] + bob_data[i];
    if (ret[i] > half)
      ret[i] = total - ret[i];
  }
}
```

Problem Statement: Compute χ^2 statistic

- Task 1b: Computing χ^2 statistic

Alice	**Bob**
Case: AA AC AA	Case: AA AC CC
Control: AA CA CA	Control: CA AC CC

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Problem Statement: Compute χ^2 statistic

- Task 1b: Computing χ^2 statistic

Alice	**Bob**
Case: AA AC AA	Case: AA AC CC
Control: AA CA CA	Control: CA AC CC

a, b: allele counts for case group
c, d: allele counts for control group
(similar to Task 1A)

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Problem Statement: Compute χ^2 statistic

- Task 1b: Computing χ^2 statistic

Alice	**Bob**
Case: AA AC AA	Case: AA AC CC
Control: AA CA CA	Control: CA AC CC

a, b: allele counts for case group
c, d: allele counts for control group
(similar to Task 1A)

$$\chi^2 = n \times \frac{(ad - bc)^2}{rsgk}$$

where $r = a + b, s = c + d, g = a + c,$
$k = b + d, n = r + s$

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Results: Compute χ^2 statistic

- Floating point computation
- Absolute accuracy
  1.11×10^{-4} with 7763 gates
  5.6×10^{-8} with 14443 gates

CODE IN ObliVM-lang: COMPUTE χ^2 STATISTIC


```
struct Task1bAutomated@n{};
float32[public n] Task1bAutomated@n.func(
    float32[public n][public 2] alice_case, float32[public n][public 2] alice_control,
    float32[public n][public 2] bob_case, float32[public n][public 2] bob_control) {
  float32[public n] ret;
  for (public int32 i = 0; i < n; i = i + 1) {
    float32 a = alice_case[i][0] + bob_case[i][0];
    float32 b = alice_case[i][1] + bob_case[i][1];
    float32 c = alice_control[i][0] + bob_control[i][0];
    float32 d = alice_control[i][1] + bob_control[i][1];
    float32 g = a + c, k = b + d;
    float32 tmp = a*d - b*c;
    tmp = tmp*tmp;
    ret[i] = tmp / (g * k);
  }
  return ret;
}
```

Building Block: Secure Set Union

Alice	**Bob**
S^A	S^B
{a, b, c}	{b, d, e}

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Cardinality of the union of the sets i.e. $|S^A \cup S^B|$

$$|S^A \cup S^B| = 5$$

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Strawman solution:

union(S^A, S^B)

1: Sort the combined array $S^A || S^B$ obliviously

$O(N \log^2 N)$

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Set Union: Oblivious Merge

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1: Local sort of S^A and S^B

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Set Union: Oblivious Merge

union(S^A, S^B)

1: Local sort of S^A and S^B
2: Oblivious merge of sorted lists
3: Compute cardinality in a single pass

$O(N \log N)$

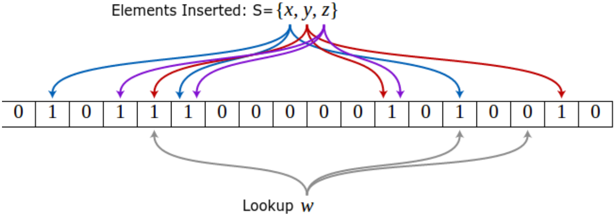
Cleartext
Secure

Code: Oblivious Merge


```
void Task2Automated@m@n.obliviousMerge(int@m[public n] key,
                                        public int32 lo,
                                        public int32 l) {
  if (l > 1) {
    public int32 k = 1;
    while (k < l) k = k << 1;
      k = k >> 1;
    for (public int32 i = lo; i < lo + l - k; i = i + 1)
      this.compare(key, i, i + k);
    this.obliviousMerge(key, lo, k);
    this.obliviousMerge(key, lo + k, l - k);
  }
}
```

Set Union: Bloom Filter

- Common case: Check for existence of elements
- Our case: Approximate the cardinality of a set S



Set Union: Bloom Filter

- Common case: Check for existence of elements
- Our case: Approximate the cardinality of a set S

$$|S|_{MLE} = \frac{\ln(1 - \frac{X}{m})}{k \ln(1 - 1/m)}$$

where
X: number of bits set,
m: number of bits in the bloom filter,
k: number of hash functions,
$|S|_{MLE}$: maximum likelihood estimate of $|S|$

Set Union: Bloom Filter

union(S^A, S^B)

1: Compute bloom filters locally

Cleartext
Secure

Set Union: Bloom Filter

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2: In secure computation, compute bitwise OR and count number of 1's

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3: Compute estimated $|S|$ in cleartext

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Set Union: Bloom Filter

union(S^A, S^B)

1: Compute bloom filters locally
2: In secure computation, compute bitwise OR and count number of 1's
3: Compute estimated $|S|$ in cleartext

$O(m)$ operations, m: number of bits used for bloom filter
$m = O(N)$, number of elements inserted in the bloom filter

Cleartext
Secure

Code: CountOnes

```
int@log(n + 1) BF_circuit.countOnes@n(int@n x) {
  if (n==1) return x;
  int@log(n - n/2 + 1) first = this.countOnes@(n/2)(x$0~n/2$);
  int@log(n - n/2 + 1) second = this.countOnes@(n - n/2)(x$n/2~n$);
  Pair<bit, Int@log(n - n/2)> ret = this.add@log(n - n/2 + 1)(first, second);
  int@log(n + 1) r = ret.right.v;
  r$log(n+1)-1$ = ret.left.v;
  return r;
}
```

Problem Statement: Hamming Distance

Alice and Bob maintain records of type (*ref*, *svtype*, *alt*) that differ from the reference

```
d = 0;
for each record of type SNP or SUB
    if ((x == null) || (y == null) || (x.ref ==
y.ref && x.alt != y.alt)
    d += 1;
end for
```

Solution: Hamming Distance

Alice	Bob
S^A ={(1, T, SNP),(75, G, SNP)}	S^B ={(1, T, SNP),(18, A, SNP)}

We need all positions that have been modified, but not modified to the same value

Hamming Distance $= |S^A \cup S^B| - |S^A \cap S^B| = |\{(75, G, SNP), (18, A, SNP)\}|$

Problem Statement: Edit Distance

Alice and Bob maintain records of type (*ref*, *svtype*, *alt*) that differ from the reference

```
Replacement:  Calculate like hamming distance
Insertion/Deletion:
    If one party modifies a position, add
len(alt) to edit distance
    If both parties modify a position, add
len(max(alt1, alt2)) to edit distance
```

Solution: Edit Distance

Alice	Bob
{(1, T, SNP), (10, TCG, INS), (75, G, SNP)}	{(1, T, SNP), (10, CA, INS), (18, A, SNP)}

Solution: Edit Distance

Alice	Bob
{(1, T, SNP), (10, TCG, INS), (75, G, SNP)}	{(1, T, SNP), (10, CA, INS), (18, A, SNP)}

$$S_1^A = \{(1,1),(10,1),(10,2),(10,3),(75,1)\}$$
$$S_2^A = \{(1,T,1),(10,T,1),(10,C,2),(10,G,3),(75,G,1)\}$$

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{(1, T, SNP), (10, TCG, INS), (75, G, SNP)}	{(1, T, SNP), (10, CA, INS), (18, A, SNP)}

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$$d1 = |S_1^A \cup S_1^B| = |\{(1,1),(10,1),(10,2),(10,3),(75,1),(18,1)\}|$$

Solution: Edit Distance

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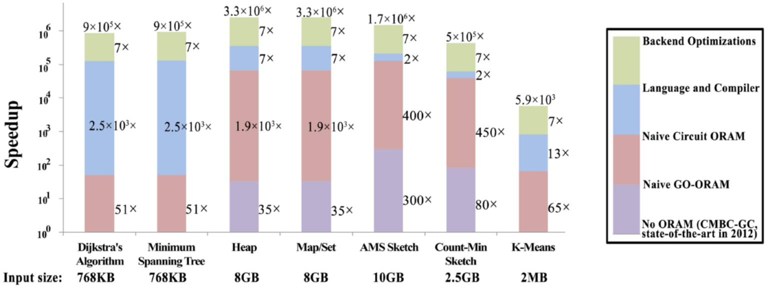
{(1, T, SNP), (10, TCG, INS), (75, G, SNP)}	{(1, T, SNP), (10, CA, INS), (18, A, SNP)}

$S_1^A = \{(1,1),(10,1),(10,2),(10,3),(75,1)\}$
$S_2^A = \{(1,T,1),(10,T,1),(10,C,2),(10,G,3),(75,G,1)\}$

$d1 = |S_1^A \cup S_1^B| = |\{(1,1),(10,1),(10,2),(10,3),(75,1),(18,1)\}|$

$d2 = |S_2^A \cap S_2^B| = |\{(1,T,1)\}|$

Compute $d1 - d2$



Thank You!

http://oblivm.com/

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